

Magnetization Currents of Fluctuative Cooper Pairs

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Recent experiments show that the Nernst-Ettingshausen effect is orders of magnitude stronger than the thermoelectric Seebeck effect in superconductors above the critical temperature. We explain different magnitudes of the two effects accounting for the magnetization current of virtual Cooper pairs. The method allows for detailed understanding of the surprising non-monotonic dependence of the Nernst-Ettingshausen coefficient on the magnetic field.

Introduction. Thermoelectric and thermomagnetic phenomena in solids were discovered in the XIXth century [1, 2]. The most significant among them are the Seebeck effect (SE), the Nernst-Ettingshausen effect (NEE) and the Ettingshausen effect (EE). The SE, also referred to as the differential thermopower, consists in the induction of the electric field in a conducting sample subjected to the gradient of temperature at zero electric current (open circuit) condition. The field is induced in the temperature gradient direction. The NEE consists in the induction of the electric field \mathbf{E} in conducting samples subjected to a magnetic field \mathbf{H} and the temperature gradient ∇T applied in the perpendicular to \mathbf{H} direction. The electric field is measured in the direction perpendicular to both magnetic field and temperature gradient in the open circuit regime and adiabatic conditions (both electric currents in the sample and the thermal flow in the direction of \mathbf{E} equal to zero, $j_x = j_y = 0$ and $q_y = 0$, respectively). In practice, the adiabatic condition is usually substituted by the isothermic one: $\nabla_y T = 0$ [3]. The EE is reciprocal to NEE: it consists in the induction of the temperature gradient $\nabla_x T$ if the current $j_y \neq 0$ propagates through the sample perpendicularly to the applied magnetic field H_z in the adiabatic conditions: $\nabla_y T = 0$ and $j_x = q_x = 0$. Due to the Onsager principle of the transport coefficients symmetry, NEE and EE usually are correlated.

The aforementioned phenomena found their theoretical explanation only in the middle of the XXth century in the works by Mott [4] and Sondheimer [5]. In a degenerate Fermi gas, thermoelectric and thermomagnetic effects were shown to be controlled by the particle-hole asymmetry, with the magnitudes of SE, NEE and EE being governed by the factor $\sim T/E_F$, where E_F is the Fermi energy. As a result, the Seebeck and Nernst-Ettingshausen coefficients for good conductors at room temperatures are of the order of $S = E_x/\nabla_x T \sim 10^{-2} \div 10^{-1} \mu\text{V}/\text{K}$ and $\nu = E_x/(H_z \cdot \nabla_x T) \sim 10^{-3} - 10^{-2} \mu\text{V}/(\text{K} \cdot \text{T})$, respectively, while they are much larger in the case of half-metals and degenerated semiconductors.

The SE in type I superconductors occurs due to the transformation of normal excitations into Cooper pairs at the edges of samples subjected to a temperature gradient

[6]. The EE which requires the magnetic field penetration in a sample can be observed only in type II superconductors and is due to the entropy transport governed by the motion of vortices. Still, the magnitude of this effect remains as small as $10^{-4} \mu\text{V}/(\text{K} \cdot \text{T})$, which is why the studies of SE and EE in superconductors had only the fundamental interest, initially.

Nowadays, the control of heat fluxes and minimization of related losses are crucially important in nanoelectronics. This is why the thermoelectric and thermomagnetic phenomena in nanostructures and new materials attracted much attention in the recent years. First indications of a sizeable EE in a wide range of temperatures in superconductors above the critical temperature were reported by Palstra *et al* [7] (see also [8, 9]) who detected it in the optimally doped YBCO samples at temperatures up to 10 K above the phase transition. The discovery of a giant EE (hundred times larger than its value in conventional metals) in the pseudogap state of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ [10] was a next milestone followed by the similar finding (with a 10^3 enhancement in magnitude in the wide range of temperatures) in the low-temperature superconductor $\text{Nb}_{0.15}\text{Si}_{0.85}$ [11]. These observations were especially surprising in view of the previously recorded data on the magnitude of the Seebeck coefficient in the fluctuative regime of superconductors, undergoing a weak singular decrease close to T_c but remaining of the same order of magnitude as in the normal phase above T_c [12–14].

The theoretical description of fluctuation contributions to the thermoelectric and thermomagnetic coefficients remains complex and controversial. Initially, the fluctuation contribution to the Seebeck coefficient in 3D superconductor was studied by Maki [15] in the framework of the time dependent Ginzburg-Landau equation, and it was found to be negligibly small. After the discovery of the anomaly in the Seebeck coefficient behavior close to T_c in monocrystals of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ [12] the problem was revisited both phenomenologically [16] and microscopically [17]. Both papers concluded that the fluctuation correction to the Seebeck coefficient S_H is proportional to the degree of particle-hole asymmetry. It logarithmically depends on temperature above T_c in the 2D case: $S_H^{(2)} \sim (T/E_F) \ln [T_c/(T - T_c)]$. In what concerns

the fluctuative EE at weak magnetic fields, it was initially studied in the framework of the GL approach in the same Ref. [16]. It was shown that the Cooper pairs contribution to the Ettingshausen coefficient does not contain the smallness induced by particle-hole asymmetry and exhibits much stronger temperature dependence than the normal phase contribution: $\beta_{yx}^{\text{fl}} \sim T_c / (T - T_c)$. After the new experimental findings of Ref. [10], the problem was revisited in Ref. [18], where the linear response theory result of Ref. [16] was reproduced and the importance of the magnetization currents was emphasized.

The magnetization currents of virtual Cooper pairs are in the focus of the Letter aimed at understanding of the surprisingly large difference in magnitudes of SE and NEE in fluctuative superconductors. These currents may be induced if the magnetization in the sample is spatially inhomogeneous. Its inhomogeneity is caused by the temperature gradient. The induced electric current contributes to the Ettingshausen coefficient (see the schematic in Fig. 1). It can be easily expressed from the Ampere law as $\mathbf{j}^{\text{mag}} = \frac{c}{4\pi} \nabla \times \mathbf{B}$, where $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$, \mathbf{H} is the spatially homogeneous external magnetic field, \mathbf{M} is the magnetization. In the presence of a temperature gradient $\nabla_x T$ one can express the magnetization current as $j_y^{\text{mag}} = -c(dM_z/dT)\nabla_x T$ [18, 19]. In the case of NEE, the open circuit condition holds: $j_x = j_y = 0$. In full analogy with a classical Hall effect, the magnetization current in y direction is compensated by the induced Nernst-Ettingshausen voltage, which yields the electric field $E_y^{\text{mag}} = \rho_{yy} j_y^{\text{mag}}$ (ρ_{yy} is the diagonal component of the resistivity tensor, $\rho_{yy} = \rho_{xx}$).

The physics of magnetization currents has been revealed half a century ago by Obratsov [19] who noticed that the Onsager principle applied to the thermoelectric tensor in the presence of magnetic fields can be fulfilled only if these currents are accounted for. In normal metals the magnetization currents are negligible so that they do not affect the classical Sondheimer results [5], obtained using the transport equation approach.

In what concerns the properties of superconductors in the fluctuation regime, Ussishikin et al. [18] demonstrated that accounting for the contribution of magnetization currents to the heat flow in the vicinity of T_c one obtains the thrice lower value of the Ettingshausen coefficient compared to what was predicted by Ullah and Dorsey [16]. The role of magnetization currents is even more important in the regime of quantum fluctuations: the Kubo-like response contribution to the heat flow [20] results in the violation of the third law of thermodynamics which can only be rectified by taking into account the fluctuative Meissner magnetization above $H_{c2}(0)$ [21]. Similar contradictions to the laws of thermodynamics were found in studies of thermo-magnetic effects in other solid state systems [22, 23], and in each case the magnetization currents contribution was crucial for resolving the paradoxes.

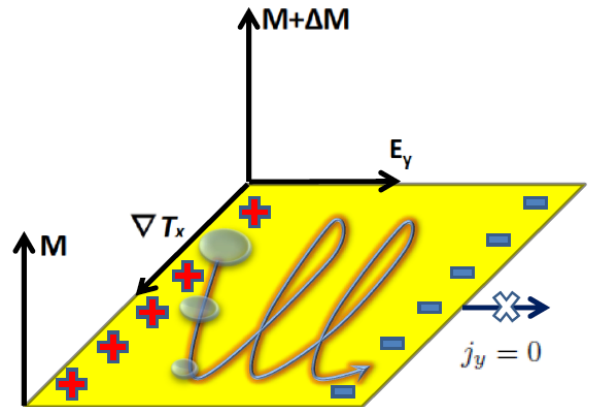


FIG. 1: Schematic representation of the FCP motion in the superconducting film subjected to the temperature gradient along x -axis. The concentration and size of FCP varies with temperature variation. The local magnetization parallel to the external magnetic field varies along x -axis as well. The spatial inhomogeneity of magnetization leads to the transformation of the FCP trajectories from circular to trochoidal ones which is why the magnetisation currents appear. To compensate these currents a voltage is induced in y -direction that provides the main contribution to NEE.

We present here a unified thermodynamic approach to NEE and SE in fluctuative superconductor which elucidates the physics behind the striking difference in their magnitudes. We show that while the Seebeck thermopower is governed by the chemical potential temperature derivative, the NE effect is dominated by magnetization currents of virtual Cooper pairs, which are present in superconductors above the critical temperature [24]. We emphasize that we consider NEE, not EE, as the open circuit condition is essential in our approach.

Generalities. Let us consider a conductor subjected to a temperature gradient and satisfying the boundary conditions: $j_x = j_y = 0$, $\nabla_y T = 0$, and $\nabla_x T \neq 0$ (see Fig. 1). As a whole, it cannot be characterized by a Fermi-Dirac distribution function as the unique temperature cannot be defined throughout the volume of the sample. On the other hand, the equilibrium distribution can be used for small enough volumes where the temperature can be assumed constant. We shall work within the *local equilibrium* approximation introducing the locally defined Fermi-Dirac function

$$f_{FD}(\varepsilon, x) = \{1 + \exp[(\varepsilon - \mu(x))/k_B T(x)]\}^{-1}, \quad (1)$$

where $\mu(x)$ and $T(x)$ are the coordinate dependent chemical potential and temperature. We underline that this approximation is not universal. In particular, it is likely to fail if the electron scattering is specifically energy dependent (Kondo effect, thermoelectric effects in

the vicinity of the $2\frac{1}{2}$ phase transition) or in the case of a strong phonon drag effect. Nevertheless, it remains a valid and powerful tool in a large variety of systems including the most part of *up-critical* (above the critical temperature T_c) superconductors. The conclusions of this Letter are restricted to the systems where the aforementioned assumptions are applicable.

Once Eq. (1) is valid, in the absence of electric current, the electro-chemical potential $\mathcal{E}(x)$ must be constant across the sample [25, 26]:

$$\mathcal{E}(x) = \mu(x) + e\varphi(x) = \text{const.} \quad (2)$$

It is instructive to apply to both parts of Eq. (2) the gradient operator, having in mind that $E_x = -\nabla\varphi$. This allows obtaining an important link between the temperature gradient and the induced electric field: $eE_x = \left(\frac{d\mu}{dT}\right) \nabla T$.

Consequently, the Seebeck coefficient writes:

$$S \equiv \frac{E_x}{\nabla T} = \frac{1}{e} \frac{d\mu}{dT}. \quad (3)$$

We will consider up-critical 2D superconductors. The coexisting subsystems of fluctuation Cooper pairs (FCP) and electrons will be assumed locally non-interacting. This means that in each small volume characterised by the coordinate x the number of electrons involved in fluctuation Cooper pairing is dependent only on $T(x)$. The FCP gas with concentration $\mathcal{N}_{\text{cp}}[T(x)]$ and the depleted electron gas with the local concentration $n_e(x) - \mathcal{N}_{\text{cp}}[T(x)]/2$ can be considered as two independent parallel conducting channels (the indices e and cp are related to electrons and FCP, respectively). Indeed, in the first order, the electron-electron interactions in the Cooper channel are taken into account once the FCP subsystem is introduced. In the second order, one would need to consider FCP interactions, but these are known to be important only in the critical vicinity of T_c .

The constancy of electrochemical potential condition applied to both electron and Cooper pair subsystems results in:

$$E_x = -\nabla\varphi_e = -\nabla\varphi_{\text{cp}} = \frac{1}{2e} \nabla\mu_{\text{cp}}. \quad (4)$$

The chemical potential of FCP needs to be derived. From its general definition one can write

$$\mu_{\text{cp}} = \frac{\partial \mathcal{F}_{\text{cp}}}{\partial \mathcal{N}_{\text{cp}}} = \frac{\partial \mathcal{F}_{\text{cp}} / \partial \epsilon}{\partial \mathcal{N}_{\text{cp}} / \partial \epsilon}, \quad (5)$$

with \mathcal{F}_{cp} being the free energy of the FCP gas, $\epsilon = \ln \frac{T}{T_c} \approx (T - T_c)/T_c$ being the reduced temperature. We shall specifically consider a superconducting film of

the thickness less than corresponding coherence length ξ . Corresponding values of $\mathcal{F}_{\text{cp}}^{(2D)}$ and $\mathcal{N}_{\text{cp}}^{(2D)}$ above the critical temperature can be written in the GL approximation as [24]

$$\mathcal{F}_{\text{cp}}^{(2D)} = -\frac{TS}{4\pi\xi^2} \epsilon \ln \epsilon; \quad \mathcal{N}_{\text{cp}}^{(2D)} = \frac{1}{4\pi\alpha\xi^2} \ln \frac{1}{\epsilon}, \quad (6)$$

where the GL parameter $\alpha = 4\pi^2/[7\zeta(3)] T_c/E_F$ is proportional to the electron-hole asymmetry factor (see Appendix A in Ref. [24]). Substitution of Eq. (6) to Eq. (5) results in [27]

$$\mu_{\text{cp}} = \alpha T \epsilon \ln \epsilon. \quad (7)$$

Seebeck coefficient. The substitution of Eq. (7) in Eq. (3) yields the Cooper pairs contribution to the Seebeck coefficient above the superconducting transition:

$$S_{\text{cp}} = \frac{1}{2e} \frac{d\mu_{\text{cp}}}{dT} = -\frac{1}{e} \frac{2\pi^2 T_c}{7\zeta(3) E_F} \ln \frac{1}{\epsilon}, \quad (8)$$

which exceeds the normal carrier contribution

$$S_e = \frac{1}{e} \frac{\partial \mu_e}{\partial T} = -\frac{\pi^2 T}{3e E_F},$$

by a large logarithmic factor. We note that Eq. (8) has a similar temperature dependence to the previously published expressions (see Ref. [15–18]) but, in contrast to them, the thermodynamic approach provides us with the explicit value of the prefactor.

Nernst-Ettingshausen (NE) signal. In a similar way, one can calculate the contribution of FCP to the NE coefficient. It is dependent on the components of the thermoelectric (β_{ij}) and resistivity (ρ_{ij}) tensors as

$$\nu = \frac{\rho_{xx}\beta_{xy} + \rho_{xy}\beta_{yy}}{B}. \quad (9)$$

Taking advantage of the relations $\beta_{yy} = -\frac{\sigma_{xx}}{2e} \frac{d\mu}{dT}$; $\beta_{xy} = c \frac{\partial M_z}{\partial T}$ derived in our previous work [28] one finds from Eq. (9) the NE coefficient of FCP as

$$\nu_{\text{cp}} = \frac{1}{2eH} \left(\frac{\sigma_{\text{cp}}^{yx}}{\sigma_{\text{cp}}^{xx}} \right) \frac{d\mu_{\text{cp}}}{dT} + \frac{c\rho_{\text{cp}}^{xx}}{H} \frac{dM_{\text{cp}}^z}{dT} = \nu_{\text{cp}}^{(\text{th})} + \nu_{\text{cp}}^{(\text{magn})}. \quad (10)$$

The fluctuation contribution to the Hall conductivity σ_{cp}^{yx} is proportional to the coefficient of the electron-hole asymmetry T/E_F (see Ref. [24]), which is why $\nu_{\text{cp}}^{(\text{th})}$ appears to be small by a parameter α^2 . On the other hand, the magnetization term $\nu_{\text{cp}}^{(\text{magn})}$ having the same singularity with respect to ϵ as $\nu_{\text{cp}}^{(\text{th})}$, does not contain the small factor α^2 . This brings us to conclusion that the fluctuation contribution to the NE coefficient is governed by the magnetization currents of FCP. The magnetization term in (10) is dependent on the resistivity of FCP, who deviates from the result of Ussishkin *et al* (Eq.(13) in Ref.

[18]) who assumed that the NE coefficient is dependent on the sum of normal phase and FCP conductivities in order to achieve a good fit to the experimental data. We argue that considering normal electrons and FCP as two parallel conductivity channels one may derive the total voltage drop as product of current and resistivity in either electronic or FCP channel. Therefore, the magnetization current of FCP must be multiplied by the resistivity of FCP only.

Close to T_c and for weak enough magnetic fields, one can use the GL approach [24]. In the most interesting 2D case, the fluctuation magnetization per unit area can be written as

$$M_{\text{cp}}^{(2D)}(\epsilon, h) = \frac{|e|T}{\pi} \left\{ \ln \frac{\Gamma(\frac{1}{2} + \frac{\epsilon}{2h})}{\sqrt{2\pi}} - \frac{\epsilon}{2h} \left[\psi(\frac{1}{2} + \frac{\epsilon}{2h}) - 1 \right] \right\},$$

while the longitudinal magnetoresistivity of FCP is given by the expression:

$$\rho_{\text{cp}}^{(2D)}(\epsilon, h) = \frac{8h^2}{e^2\epsilon} \left[\psi(\frac{1}{2} + \frac{\epsilon}{2h}) - \psi(\frac{\epsilon}{2h}) - \frac{h}{\epsilon} \right]^{-1}.$$

In the above expressions, $\psi(z)$ is the logarithmic derivative of the Euler Gamma function $\Gamma(z)$, $h = 2\pi\xi^2 H/\Phi_0$ is the dimensionless magnetic field. Substituting these expressions into Eq. (10) one finds for the NE signal ($N = \nu H$):

$$\begin{aligned} N_{\text{cp}}^{(2D)}(\epsilon, h) &= \left(\frac{4}{|e|} \right) \frac{\frac{\epsilon}{2h} \psi'(\frac{1}{2} + \frac{\epsilon}{2h}) - 1}{1 - \frac{\epsilon}{h} [\psi(\frac{1}{2} + \frac{\epsilon}{2h}) - \psi(\frac{\epsilon}{2h})]} \\ &= \frac{8}{3|e|} \left(\frac{h}{\epsilon} \right) \begin{cases} 1, & h \ll \epsilon \\ \frac{3\epsilon}{2h}, & h \gg \epsilon \end{cases}. \end{aligned} \quad (11)$$

Eq. (11) reproduces both the giant value of the fluctuation NE signal observed in numerous experiments (as compared to $N_e = -\pi^2 T \tau H / (6m_e c E_F)$) and its linear increase as a function of the magnetic field in weak enough fields ($h \ll \epsilon$). However, the saturation of the NE signal at the fields $h \gtrsim \epsilon$ predicted by the GL model does not find its confirmation in the experimental data. Quite contrarily, the experiments show that both conventional and unconventional superconductors demonstrate the characteristic maximum of the NE signal at $h \sim \epsilon$ [29, 30]. The maximum in the magnetic field dependence of the NE signal persists at $T \gg T_c$ [29], i.e. far beyond the GL model range of validity.

It worth to recall that the similar problem was discussed in 1970's in relation to the formal saturation of the fluctuation magnetization of 2D superconductors in strong fields calculated using the GL model [31]. This seeming paradox was explained by the early breakdown of the GL scenario at relatively weak magnetic fields where the magnetic length of a Cooper pair approaches the GL coherence length $\xi_{GL}(\epsilon)$. Interestingly, this happens at

$h \sim \epsilon$, where the minimum in magnetization and the maximum in the NE signal magnetic field dependencies are expected.

Ref. [32] shows that the short wave and dynamic fluctuation modes must be taken into account when calculating the magnetization and conductivity of fluctuative superconductors. In full generality, the fluctuation part of the free energy can be represented as the trace of the logarithm of the fluctuation propagator [24]

$$\mathcal{F}_{\text{cp}}^{(2D)}(T, H) = -\frac{|e|H}{\pi} T \sum_k \sum_n \ln [g L_n^{-1}(\Omega_k)], \quad (12)$$

where g is the effective BCS interaction strength, while the fluctuation propagator $L_n(\Omega_k)$ is the two particle Green function describing fluctuation Cooper pairings of electrons in a wide range of temperatures above the line $T_c(H)$ [24]:

$$L_n^{-1}(\Omega_k) = -\zeta \mathcal{E}_n(\Omega_k).$$

Here ζ is the electron density of states and

$$\mathcal{E}_n(\Omega_k) = \ln \frac{T}{T_{c0}} + \psi \left[\frac{1 + |k|}{2} + \frac{|e|\mathcal{D}H}{\pi c T} \left(n + \frac{1}{2} \right) \right] - \psi \left(\frac{1}{2} \right),$$

where \mathcal{D} is the electron diffusion coefficient. The summation in Eq. (12) is performed over the Landau levels n and corresponding bosonic frequencies of Cooper pairs $\Omega_k = 2\pi T k$.

According to Eq. (10), the magnetization current of fluctuating Cooper pairs now can be expressed through the second derivative of the free energy (12). One can notice that the summation of the terms in Eq. (12) containing $\ln(g\zeta)$ yields temperature and magnetic field independent constants which do not contribute to the NE signal. As a result, one can find the general expression for the NE signal as:

$$N_{\text{cp}}^{(2D)}(T, H) = \frac{c\rho_{\text{cp}}^{xx}}{\Phi_0} \frac{d}{dT} \frac{\partial}{\partial H} \left[HT \sum_k \sum_n \ln \mathcal{E}_n(\Omega_k) \right]. \quad (13)$$

This expression allows finding $N_{\text{cp}}^{(2D)}(T, H)$ in the whole range of magnetic fields and temperatures above the phase boundary $H_{c2}(T)$.

In particular, one can derive analytically the magnetic field and temperature dependencies of the NE signal at very low temperatures close to the second critical field $H_{c2}(0)$, where only the contribution of the lowest Landau level is essential and summation over bosonic frequencies can be done exactly. The corresponding expression for $M_{\text{cp}}^{(2D)}(T, \hat{h})$ can be found in Ref.[33]. Of interest for us is its temperature derivative which has a form:

$$\frac{dM_{\text{cp}}^{(2D)}(T, \tilde{h})}{dT} = \frac{|e|}{\pi\gamma_E} \left\{ \left(\frac{16\gamma_E^3 T^2}{\pi^2 \tilde{h} T_{c0}^2} - 1 \right) \left[\frac{\gamma_E}{\tilde{h}} - \frac{\tilde{h} T_{c0}^2}{2\gamma_E T^2} \psi' \left(\frac{\tilde{h} T_{c0}}{2\gamma_E T} \right) \right] - \frac{T_{c0}}{T} \right\}. \quad (14)$$

$\gamma_E = 1.78$. Here we took into account also the temperature dependence of the second critical field:

$$H_{c2}(T) = \frac{\pi T_{c0}}{2\gamma_E \mathcal{D}} \frac{c}{|e|} \left(1 - \gamma_E^2 \frac{T^2}{T_{c0}^2} \right).$$

The Cooper pair contribution to the resistivity above T_c can be found in Ref. [33, 34]:

$$\rho_{\text{cp}}^{(2D)}(T, \tilde{h} \ll 1) = \left\{ \begin{array}{ll} \frac{\pi^2}{2\gamma_E e^2} \frac{\tilde{h} T_{c0}}{T}, & \tilde{h} \ll \frac{T}{T_{c0}} \\ \frac{3\pi^2}{2e^2 \ln \tilde{h}}, & \frac{T}{T_{c0}} \ll \tilde{h} \end{array} \right|. \quad (15)$$

with $\tilde{h} = [H - H_{c2}(0)] / H_{c2}(0)$. Eqs. (14) and (15) allow us to obtain the asymptotic behavior of the Nernst signal in the low temperature range:

$$N_{\text{cp}}^{(2D)}(T, \tilde{h}) = \frac{\gamma_E}{|e|} \left(\frac{T}{\tilde{h} T_{c0}} \right) \left\{ \begin{array}{ll} \frac{8\gamma_E}{\pi}, & \tilde{h} \ll \frac{T}{T_{c0}} \\ -\frac{1}{\tilde{h} \ln \frac{1}{\tilde{h}}}, & \frac{T}{T_{c0}} \ll \tilde{h} \end{array} \right|.$$

The temperature dependence of the maximum in the NE signal dependence on the magnetic field is of special interest. Recently, the authors of Ref. [29, 30] have proposed using it for the precise determination of the second critical field $H_{c2}(0)$, often inaccessible for direct measurements because of its huge value. The analysis of the experimental data obtained on the HTS compound $\text{Pr}_{2-x}\text{Ce}_x\text{CuO}_4$ led the authors of Ref. [29] to propose a phenomenological expression:

$$H_{\text{max}}^*(T) = H_{c2}(0) \ln \frac{T}{T_{c0}}. \quad (16)$$

Our Eq. (13) unfortunately does not allow to extract analytically the temperature dependence of interest, $H_{\text{max}}^*(T)$. Nevertheless, due to the specific scaling form of Eq. (13) the temperature dependence of the magnetic field corresponding to the maximum of the Nernst signal can be expressed in the generic form:

$$H_{\text{max}}^* \left(\frac{T}{T_{c0}} \right) = \frac{T}{T_{c0}} \varsigma \left(\ln \frac{T}{T_{c0}} \right), \quad (17)$$

where $\varsigma(x)$ is some smooth function which satisfies the condition $\varsigma(0) = 0$.

We note that Eq. (17) coincides with Eq. (16) only in the particular case of $\varsigma(x) = x \exp(-x)$. In the case of any other analytical function $\varsigma(x)$, the magnetic field corresponding to the maximum of the NE signal, $H_{\text{max}}^*(T)$, would increase linearly with the increase of temperature.

The heuristic justification of Eq. (16) is based on the statement that the maximum in the NE signal magnetic field dependence occurs where the FCP size $\xi_{GL}(T)$ is of the order of its magnetic length $\ell_{H_{\text{max}}^*} = (c/|e|H_{\text{max}}^*)^{1/2}$ [11, 29, 30]. Close to the critical temperature, this indeed yields $H_{\text{max}}^* \sim H_{c2}(0) (T - T_{c0}) / T_{c0}$. Far from T_{c0} , the authors of [11, 29, 30] extend the GL expression as $\xi_{GL}(T) = \xi_{BCS} / \sqrt{\ln \frac{T}{T_{c0}}}$, which brings them to Eq. (16). We believe that this extension lacks justification, and the rigorous expression (13) needs to be used, in the general case.

In conclusion, we have derived the Seebeck and Nernst-Ettingshausen coefficients for fluctuative superconductors, in the local equilibrium approximation. A thermodynamical approach allows analytical evaluation of both constants which appear to be different by orders of magnitude due to the crucial contribution of magnetization currents to the NE signal. We explain the non-monotonous behaviour of the NE signal as a function of magnetic field above T_c and estimate the position of the maximum of this dependence from a simple scaling argument.

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